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SIMPLE WAVES AND STRONG DISCONTINUITIES IN A MAGNETIZABLE MEDIUM

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It is shown that in a compressible conducting medium magnetizable according to an arbitrary isotropic law $\mu = \mu(\rho, T, H)$ (μ and ρ are the magnetic permeability and density of the medium, T is the temperature, and H is the magnetic field intensity) exist simple waves of the same type as in a nonmagnetic medium. A plane polarized simple magnetohydrodynamic wave exists in a magnetizable incompressible fluid in addition to entropy and Alfvén waves. Equations of Alfvén simple waves are integrated.

Discontinuities in a magnetizable medium are classified. In the case of a conductive medium the system of conditions at discontinuities, except rotational and plane polarized ones, admit nonpolarized discontinuities with change of thermodynamic parameters and of the magnetic induction vector (as to magnitude and direction). It is shown that in a nonconducting compressible medium magnetizable in conformity with the law $\mu = \mu(H)$ the shock waves are gasdynamic.

1. The propagation of one-dimensional perturbations in a nonuniformly and isotropically magnetizable conduction medium is defined by the system of equations

$$\frac{\partial u_i}{\partial t} + x_{ik}\frac{\partial u_k}{\partial x} = 0 \tag{1.1}$$

where $u_1 = \rho'$, $u_2 = s'$, $u_3 = v_x'$, $u_4 = v_y'$, $u_5 = v_z'$, $u_6 = B_y'$ and $u_7 = B_z'$; ρ', s', ... are perturbations of magnetohydrodynamic variables.

Let us, first, consider a compressible medium whose equations of state in the absence of an electromagnetic field are of the form

$$T = T(\rho, s), \quad p = p(\rho, s)$$

The nonzero elements of matrix $\{x_{ik}\}$ for this case appear in [1].

Seeking the solution of system (1.1) in the form of plane waves $u_i = u_i^0 \exp i\cdot(kx - \omega t)$, we obtain for the determination of phase velocities $\lambda \equiv \omega / k$ the dispersion equation

$$\lambda \det (X_1 A + X_6 B - \lambda^2 E) = 0 \tag{1.2}$$

$$X_i = \begin{Vmatrix} x_{3i} & x_{3,i+1} \\ x_{4i} & x_{4,i+1} \\ x_{5i} & x_{5,i+1} \end{Vmatrix}, \quad A = \begin{Vmatrix} \rho & 0 & 0 \\ x_{23} & x_{24} & x_{25} \end{Vmatrix}, \quad B = \begin{Vmatrix} B_y & -B_x & 0 \\ B_z & 0 & -B_x \end{Vmatrix}$$

where $\lambda = 0$ is a simple root of Eq. (1.2) that determines the entropy wave, and E is a unit matrix. It can be shown that for $\lambda^2 = B_x^2 / (4\pi\rho\mu)$ the last rows of the determinant in the left-hand side of this equation are proportional for any arbitrary magnetization law of the form $\mu = \mu(\rho, T, H)$. Hence the Alfvén velocity satisfies the most general dispersion equation for magnetizable compressible media.

The remaining phase velocities are determined by the biquadratic equation

$$\lambda^4 - 2c_1\lambda^2 + c_2 = 0 \tag{1.3}$$

$$2c_1 = \rho x_{31} + x_{23}x_{32} + B_y x_{36} + B_z x_{37} + x_{24}x_{42} + x_{25}x_{52} + mB_x^2 (\mu^2 + \mu_H B_x^2 / B)$$

$$2c_1 - c_2 = [\rho x_{41} + x_{23}x_{42} + B_y x_{46} + B_z x_{47}] [x_{32}x_{24} - B_x x_{36}] \frac{B_y^2 + B_z^2}{B_y^2}$$

Consequently, as in the case of a nonmagnetic medium (μ = const), there are seven types of simple waves in a medium that is magnetizable in conformity with an arbitrary isotropic law.

Variation of magnetohydrodynamic variables in a simple wave is defined by the following system of nonlinear differential equations [2] :

$$du_i / du_m = r_i / r_m \tag{1.4}$$

where r_k are components of the right-hand eigenvector of matrix $\{x_{ik}\}$. After the determination of u_i as functions of u_m using system (1.4), the function $u_m(x, t)$ in the case of a simple wave can be obtained from the equation

$$x - \lambda_k (u_1, u_2, \ldots, u_7) t = F(u_m) \tag{1.5}$$

in which function F is determined by the input conditions.

2. Let us consider the types of one-dimensional Riemann waves in a magnetizable medium.

When the magnetic field orientation is arbitrary, the right-hand vector for the Alfvén wave is of the form

$$\mathbf{r} = (0, 0, 0, \lambda B_z, -\lambda B_y - B_x B_z, B_x B_y), \qquad \lambda = \pm B_x / \sqrt{4\pi\rho\mu}$$

Substituting the variable v_y for u_m from (1.4) we obtain

$$\frac{dv_z}{dv_y} = -\frac{B_y}{B_z}, \quad \frac{dB_z}{dv_y} = \frac{B_y B_x}{\lambda B_z}, \quad \frac{dB_y}{dv_y} = -\frac{B_x}{\lambda} \tag{2.1}$$

$$d\rho = ds = dv_x = 0$$

This shows that in an Alfvén wave $B_y^2 + B_z^2$, ρ, v_x, s (and T) remain unchanged so that, also, here that wave has a circular polarization, and its longitudinal velocity and thermodynamical parameters, as well as the magnetic permeability do not change. This follows from the magnetization law $\mu = \mu(\rho, T, B / \mu)$ under condition that it is solvable for μ, i.e. $\mu^2 + \mu_H B \neq 0$, which is usually satisfied.

Hence it follows from (2.1) that

$$v_y = \mp \frac{B_y}{\sqrt{4\pi\rho\mu}} + \text{const}, \qquad v_z = \mp \frac{B_z}{\sqrt{4\pi\rho\mu}} + \text{const}$$

In this case formula (1.5) assumes the form

$$x - (v_x \pm B_x / \sqrt{4\pi\rho\mu})\, t = F(v_y)$$

which shows that the simple Alfvén wave propagates without altering its shape.

Simple magneto-sonic waves have the following phase velocities (see (1.3)):

$$\lambda_\pm = c_1 \pm \sqrt{c_1^2 - c_2} \tag{2.2}$$

where the upper sign relates to a fast and the lower to a slow magnetosonic waves, respectively. It can be shown that the relations

$$r_{5,\pm} = r_{4,\pm} \frac{B_z}{B_y}, \quad r_{7,\pm} = r_{6,\pm} \frac{B_z}{B_y} \tag{2.3}$$

hold for right-hand components of eigenvectors of corresponding magnetosonic waves.

We now obtain

$$\frac{dB_z}{dB_y} = \frac{r_{7,\pm}}{r_{6,\pm}} = \frac{B_z}{B_y}, \quad \frac{dv_z}{dv_y} = \frac{r_{5,\pm}}{r_{4,\pm}} = \frac{B_z}{B_y} \tag{2.4}$$

so that $B_z / B_y = \text{const}$, $v_z / v_y = \text{const}$, and $v_z - v_y B_z / B_y = \text{const}$.

Consequently in a magnetizable medium magneto-sonic waves are plane-polarized. Setting in (2.4) the constants of integration equal zero, we obtain $B_z \equiv 0$, and $v_z \equiv 0$. Hence in Eqs. (1.4) for magneto-sonic waves it is possible to use for components of the right-hand eigenvectors formulas (10) of [3]. After some transformations using (1.3) the equtions of simple magneto-sonic waves can be written in the form

$$\frac{dv_x}{d\rho} = \frac{\lambda_\pm}{\rho}, \quad \frac{ds}{d\rho} = \frac{NL_\pm - (L_0 + L_1)\, N\mu\mu_T m^2 B_x^2 B_y^2}{\rho\,(\lambda_\pm^2 - L_0 m B_x^2)} \tag{2.5}$$

$$\frac{dv_y}{d\rho} = -\frac{\lambda_\pm m B_x B_y (L_0 + L_1)}{\rho\,(\lambda_\pm^2 - L_0 m B_x^2)}, \quad \frac{dB_y}{d\rho} = \frac{B_y (\lambda_\pm^2 + L_1 m B_x^2)}{\rho\,(\lambda_\pm^2 - L_0 m B_x^2)}$$

where

$$L_0 = \mu^2 + \mu_H B_x^2 / B + N T_s \mu^2 \mu_T^2 m B_y^2$$

$$L_1 = N\rho\mu\,[\mu_T (s_T^e T_s - T_\rho) - \mu_\rho (1 + s_T^e T_s)]$$

$$L_\pm = (\lambda_\pm^2 - L_0 m B_x^2)\,[\mu_T \rho m B^2 (\mu_\rho + \mu_T T_\rho) - \mu\mu_T m B_y^2 - \rho\,(s_\rho^e + s_T^e T_\rho)]$$

$$N = [1 + T_s (s_T^e - \mu_T^2 m B^2)]^{-1}$$

$$m = [4\pi\rho\mu\,(\mu^2 + \mu_H B)]^{-1}, \qquad s^e = \frac{1}{4\pi\rho} \int_0^H \mu_T H \, dH$$

For magneto-sonic waves from formula (1.5) we obtain

$$x - (v_x \pm \lambda_\pm)\, t = F(\rho)$$

Since here v_x and $\lambda_\pm$ vary simultaneously with density ρ, the profile of these waves becomes deformed in time.

Let us now consider the entropy wave which does not propagate relative to the medium. For the corresponding components of the right-hand vector in this wave we have $r_7 = (B_z / B_y)\, r_6$.

Thus such wave is also plane-polarized, and it is possible to select a system of coordinates such that $B_z \equiv 0$. Using formula (9) of [3] it is possible to write the differential equation of the simple entropy wave in the form

$$K \frac{ds}{d\rho} = -[K_1 m B_x (\mu^2 + \mu_H B_x^2/B) + K_2 \rho^2 \mu_\rho (\mu^2 + \mu_H B) m B_x^3] \qquad (2.6)$$

$$K \frac{dB_y}{d\rho} = -\mu m B_x B_y [K_1 \mu_T T_s - K_2 (p_s + \psi_T T_s)]$$

$$v_x = \text{const}, \quad v_y = \text{const}, \quad v_z = \text{const}, \quad B_z \equiv 0$$

$$K = (p_s + \psi_T T_s)(\mu^2 + \mu_H B_x^2 / B) m B_x + \rho^2 \mu_\rho \mu_T T_s (\mu^2 + \mu_H B) m^2 B_x^3$$

$$K_1 = p_\rho + \psi_\rho + \psi_T T_\rho,$$

$$K_2 = \mu_\rho + \mu_T T_\rho, \qquad \psi = \frac{1}{4\pi} \int_0^H H (\mu - 1 - \rho \mu_\rho) \, dH$$

Thus for the entropy wave in a magnetizable medium not only variation of entropy is characteristic, as in the case of nonmagnetic medium, but also variation of density, induction, and temperature.

3. One-dimensional motions of a magnetizable incompressible $(\rho = \text{const})$ fluid are defined by five magnetohydrodynamic variables

$$u_1 = T, \quad u_2 = v_y, \quad u_3 = v_z, \quad u_4 = B_y, \quad u_5 = B_z$$

In the absence of a field the equation of state of the fluid is taken as $T = T(s)$ [4].

Five types of simple waves exist in an incompressible conducting magnetizable fluid. Their phase velocities are determined by the equation

$$\lambda (\lambda^2 - A_x^2)(\lambda^2 - G_x^2) = 0$$

$$G_x^2 = \frac{A_x^2}{\mu^2 + \mu_H B} \left[\mu^2 + \mu_H \frac{B_x^2}{B} + N \mu^2 \mu_T^2 T_s m (B_y^2 + B_z^2) \right]$$

For the Alfvén wave $\lambda = \pm A_x = \pm B_x / \sqrt{4\pi\rho\mu}$ and the corresponding right-hand eigen vector is

$$\mathbf{r} = (0, \lambda B_z, -\lambda B_y, -B_x B_z, B_x B_y)$$

The differential equations of a simple Alfvén wave in an incompressible fluid are the same as Eqs. (2.1) for an incompressible medium.

A simple magnetohydrodynamic transverse wave propagates relative to the fluid at phase velocity $\lambda = \pm G_x$, to which corresponds the right-hand eigen-vector with components $r_1 = N \mu \mu_T T_s m B_x (B_y^2 + B_z^2)$, $r_2 = \lambda B_y$, $r_3 = \lambda B_z$, $r_4 = -B_x B_y$ and $r_5 = -B_x B_z$. Thus this wave, unlike the Alfvén wave is plane-polarized. Moreover in it $v_z - v_y B_z / B_y = \text{const}$. Hence it is possible to set $v_z \equiv 0$ and $B_z \equiv 0$.

The differential equations of the transverse magnetohydrodynamic wave is of the form

$$dv_y / dB_y = -\lambda / B_x, \quad dT / dB_y = -N \mu \mu_T T_s m B_y$$

This system of equations was previously obtained in [5] for a conducting fluid magnetized to saturation. It reduces to quadratics for an arbitrary magnetization law of the form $\mu = \mu(H)$.

Finally, in a simple entropy wave $\lambda = 0$ and the corresponding right-hand eigenvector

$$\mathbf{r} = (\mu^2 + \mu_H B_x^2 / B, 0, 0, \mu\mu_T B_y, \mu\mu_T B_z)$$

This shows that this wave is also plane-polarized. Its equation can be written as

$$dT / dB_y = (\mu^2 + \mu_H B_x^2 / B) / (\mu\mu_T B_y), \quad v_y = \text{const}$$
$$v_z = \text{const}, \quad B_z \equiv 0$$

4. The above analysis shows that the deformation of simple wave profiles in a compressible medium can only induce the same kind of strong discontinuities as in nonmagnetic medium.

Let us consider the types of possible solutions for conditions at strong discontinuities, taking first the conducting magnetizable medium. The following system of equations defining strong discontinuities in such medium was obtained in [6].

$$\langle \rho v_n \rangle = 0, \quad \langle \rho v_n \mathbf{v}_\tau - \mu H_n \mathbf{H}_\tau \rangle = 0 \tag{4.1}$$
$$\langle \rho v_n^2 + p - \rho^2 u_\rho - (4\pi)^{-1} \mu H_n^2 \rangle = 0$$
$$\langle \rho v_n (v^2 / 2 + W - u - \rho u_\rho + T u_T) +$$
$$(4\pi)^{-1} (v_n \mu H^2 - \mu H_n (\mathbf{vH})) \rangle = 0$$
$$\langle \mu H_n \rangle = 0, \quad \mu H_n \langle \mathbf{v}_\tau \rangle = \langle v_n \mu \mathbf{H}_\tau \rangle, \quad \langle \mathbf{H}_\tau \rangle = \frac{4\pi}{c} [\mathbf{i} \times \mathbf{n}]$$
$$u = (4\pi\rho)^{-1} \int_0^H \mu(\rho, T, H)\, H\, dH, \quad u_\rho \equiv \frac{\partial u}{\partial \rho}, \quad u_T \equiv \frac{\partial u}{\partial T}$$

where subscripts n and τ denote, respectively, the normal and tangent vector components relative to the discontinuity surface, angled brackets denote jumps of quantities in the shock wave, W is the enthalpy of the medium in the absence of a field, and the remaining notation is conventional.

System (4.1) generally admists the following types of discontinuities:

1°. Discontinuities without flow of matter through the surface ($\rho v_n = m_n = 0$),

1) contact discontinuities for which $B_n \neq 0$,

2) trangential discontinuities ($B_n = 0$).

2°. Discontinuities with flow of matter through the surface ($m_n \neq 0$),

1) nonpolarized discontinuities with thermodynamic parameters and the tangential components of magnetic induction and velocity vectors varying (as to magnitude and direction) when passing through these,

2) rotational discontinuities along which the magnetic induction is continuous, and the induction vector and, generally, thermodynamic variables vary,

3) plane-polarized shock waves at passage through which thermodynamic variables become discontinuous, tangential components of magnetic induction and velocity vectors lie in one plane and vary in magnitude.

Discontinuities without flow of matter were analyzed in detail in [6].

Let us consider discontinuities with flow of matter through the surface. In this case system (4.1) may be written in the form

$$\left\langle p(\rho, T) - \rho^2 u_\rho + \frac{1}{\rho}\left(m_n^2 - \frac{\rho B_n^2}{4\pi\mu}\right)\right\rangle = 0 \quad (4.2)$$

$$\left\langle W(\rho, T) + T u_T - u - \rho u_\rho + \frac{m_n^2}{2\rho^2} + \frac{\mu H_\tau^2}{4\pi\rho}\left(1 - \frac{\rho B_n^2}{8\pi\mu m_n^2}\right)\right\rangle = 0$$

$$\langle B_n \rangle = 0, \quad \langle m_n \rangle = 0, \quad \langle B_n \mathbf{v}_\tau - v_n \mathbf{B}_\tau \rangle = 0$$

$$\left\langle m_n \mathbf{v}_\tau - \frac{B_n \mathbf{H}_\tau}{4\pi}\right\rangle = 0$$

When $B_n \neq 0$ it is possible to select the coordinate system so that on both sides of the discontinuity

$$\mathbf{v}_\tau = \frac{v_n}{B_n}\mathbf{B}_\tau$$

From the last of formulas (4.2) we then obtain

$$\left\langle \frac{\mu \mathbf{H}_\tau}{\rho}\left(m_n^2 - \frac{\rho B_n^2}{4\pi\mu}\right)\right\rangle = 0 \quad (4.3)$$

The first two of relations (4.2) together with (4.3) make it, thus, possible to obtain ρ_2, T_2, and H_2 as functions of ρ_1, T_1, and H_1 with constants m_n and B_n.

Since in nonpolarized jumps vectors $\mathbf{B}_{\tau 1}$ and $\mathbf{B}_{\tau 2}$ are neither parallel nor equal and $\rho_1 \neq \rho_2$, hence, as implied by (4.3), we have

$$v_{ni} = \frac{B_n^2}{4\pi\rho_i\mu_i} \quad (i = 1, 2) \quad (4.4)$$

and, consequently, the velocity of the medium is the same on both sides of discontinuity and equal to the Alfvén velocity. For the calculation of such discontinuities from (4.2) and (4.3) we have

$$\left\langle \frac{\mu}{\rho}\right\rangle = 0, \quad \langle p - \rho^2 u_\rho \rangle = 0 \quad (4.5)$$

$$\langle W - u - \rho u_\rho + T u_T + \mu H_\tau^2 / (8\pi\rho) \rangle = 0$$

Such waves do not exist in nonmagnetic medium, since for $\mu = 1$ from (4.5) we have

$$\langle \rho \rangle = 0, \quad \langle p \rangle = -\frac{1}{8\pi}\langle H_\tau^2 \rangle, \quad \langle W \rangle = -\frac{1}{8\pi\rho}\langle H_\tau^2 \rangle$$

which is only possible when $\langle \rho \rangle = \langle p \rangle = \langle W \rangle = 0$ and, consequently such wave becomes an Alfvén discontinuity.

For rotational jumps (vectors $\mathbf{B}_{\tau 1}$ and $\mathbf{B}_{\tau 2}$ are of equal magnitude but not parallel and $\rho_1 \neq \rho_2$) from (4.2) we have the relations (4.4) and (4.5) with the additional condition $\langle B_\tau \rangle = 0$.

A particular case of rotational discontinuities is that of Alfvén discontinuities (vectors $\mathbf{B}_{\tau 1}$ and $\mathbf{B}_{\tau 2}$ are of equal magnitude but not parallel and $\rho_1 = \rho_2$) for which from (4.4) and (4.5) we obtain

$$v_{n_1}^2 = v_{n2}^2 = B_n^2 / (4\pi\rho\mu), \quad \langle \mu \rangle = \langle H \rangle = 0$$

If then μ is a single-valued function of temperature, which is usually always assumed, also $\langle T \rangle = \langle p \rangle = \langle W \rangle = 0$.

It will be seen that when $\mu = \mu(H)$ and μ is a single-valued function of the magnetic function $(\mu^2 + \mu_H B \neq 0)$, then from condition $\langle B \rangle = 0$ ' for rotational waves we obtain $\langle\mu\rangle = 0$. It then follows from (4.5) that in such magnetizable medium rotational discontinuities other than the Alfvén ones are impossible.

Numerical analysis of system (4.5) shows that for a perfect gas magnetizable in conformity with the Klausius – Mosotti law $((\mu - 1)\, T\,/\,(\rho\mu) = \text{const})$ in the region of magnetic permeability $1 < \mu < 2$ up to the discontinuity and for the adiabatic exponent $1 < \gamma < 2$ $(\gamma = c_p/\,c_v)$ at stable, in the meaning of [3], states of the medium, only Alfvén discontinuities are possible among rotational and nonpolarized discontinuities at which entropy does not decrease.

It can be similarly shown that in a perfect gas magnetized to saturation and constant magnetization $M = (\mu - 1)\, H\,/\,(4\pi) = \text{const}$ there exist solutions which correspond to nonpolarized discontinuities and satisfy the condition of entropy increase. However these solutions are nonevolutionary.

Since for the arbitrary magnetization law $\mu = \mu\,(\rho,\, T,\, H)$ the addition of condition $\langle B_\tau\rangle = 0$ to system (4.5) reduces the number of variables that are to be determined, that condition imposes additional links on parameters m_n and $B^2 = B_\tau^2 + B_n^2$. Hence a rotational discontinuity with a jump of thermodynamic variables can only exist at specific values of the field. This, apparently, indicates that among rotational and nonpolarized discontinuities only Alfvén ones can be realized.

The question of existence of nonpolarized discontinuities that satisfy the conditions of entropy increase, and of medium stability and evolution in the case of arbitrary equations of state and magnetization laws remains open.

The plane-polarized jumps represent the most general case. The complete system (4.2), (4.3) is used for their calculation. The equation of shock adiabat may be used in the case of magnetizable media [6].

The system of conditions at strong discontinuities in an incompressible magnetizable conducting fluid admits the same types of discontinuities; rotational discontinuities can only be of the Alfvén type. When a conducting fluid magnetizes in conformity with the law $\mu = \mu\,(H)$, then only Alfvén and plane-polarized discontinuities are possible in it.

Note that because magneto-sonic simple waves are plane-polarized and in a simple Alfvén wave hydrodynamic parameters do not vary, the intensity of nonpolarized shock waves can only be finite [8]. Such discontinuities may occur, for instance, under interaction of other types of discontinuities (*).

5. There are seven types of simple waves in a nonconducting magnetizable medium [9] which are plane-polarized. Hence only plane-polarized shock waves are possible in a nonconducting magnetizable medium. For their determination we have the system of equations

$$\begin{gathered}\langle m_n\rangle = 0, \quad \langle m_n^2/\rho + p - \rho^2 u_\rho - (4\pi)^{-1}\mu H_n^2\rangle = 0 \qquad (5.1)\\ \langle m_n^2\,/\,(2\rho^2) + W - u - \rho u_\rho + T u_T\rangle = 0\\ \langle \mathbf{v}_\tau\rangle = 0, \quad \langle \mathbf{H}_\tau\rangle = 0, \quad \langle B_n\rangle = 0\end{gathered}$$

*) Strong discontinuities were also considered in the paper by V. V. Gogosov, N. L. Vasil'eva, N. G. Taktarov, and G. A. Shaposhnikov, Equations of hydrodynamics of polarizable and magnetizable multicomponent and multiphase media. Discontinuous solutions. Investigation of discontinuous solutions with a jump of magnetic permeability. Otchet Inst. Mekhaniki MGU, No. 1705, 1975.

T h e o r e m. Shock waves in a nonconducting medium that magnetizes in conformity with the law $M = M(H)$ degenerate in gasdynamic waves when conditions $dM / dH > 0$ and $d^2M / dH^2 \leqslant 0$ $(\mu > 1)$ or $dM / dH < 0$ and $d^2M / dH^2 \geqslant 0$ $(\mu < 1)$ are satisfied.

These conditions for the law of magnetization are inherent to all known media. This law shows that in paramagnetic and diamagnetic materials the following inequalities are satisfied:

$$0 \geqslant \mu_H \geqslant \frac{1-\mu}{H} \quad (\mu > 1), \quad 0 \leqslant \mu_H \leqslant \frac{1-\mu}{H} \quad (\mu < 1) \tag{5.2}$$

When $\mu = \mu(H)$ from (5.1) we obtain

$$\langle m_n \rangle = 0, \quad m_n^2 \langle 1 / (2\rho^2) \rangle + \langle W \rangle = 0 \tag{5.3}$$

$$\langle p \rangle + m_n^2 \langle 1 / \rho \rangle = \left\langle \frac{B_n^2}{4\pi\mu} - \frac{1}{4\pi} \int\limits_0^H \mu(H) H \, dH \right\rangle = Q, \quad \langle B_n \rangle = 0, \quad \langle \mathbf{H}_\tau \rangle = 0$$

To prove the theorem we take into account that the law of magnetization $\mu = \mu(H) = \mu(\sqrt{H_\tau^2 + B_n^2 / \mu^2})$ by virtue of the last three of conditions (5.3) is an implicit equation in μ with parameters H_τ and B_n, whose condition of solvability is $1 + \mu_H B_n^2 / (\mu^3 H) \neq 0$. In the class of positive functions of μ, when condition (5.2) is satisfied, it applies to paramagmetic as well as to diamagnetic materials. Hence from conditions (5.3) we have $\langle \mu \rangle = 0$ $\langle H^2 \rangle = B_n^2 \langle 1 / \mu^2 \rangle = 0$, and $Q = 0$.

Thus (5.3) reduce to gasdynamic conditions and to conditions for a field which does not interact with the medium. Since in the case of media in which magnetization is independent of temperature and the adiabat $\langle s + s^e \rangle = 0$ is the same as in gasdynamics $(\langle s \rangle = 0)$, solutions at shock waves (5.3) coincide with those in gasdynamics.

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